1 Revision of Arithmetic and Algebra

1.1 The Number Line: Order and Inequalities

1.1.1 Revision of Number Sets

The very foundation of math was always to combine numbers in some way to achieve a desired result. Results were obtained by the application of an operator to a number, such as 3 + 4 =7. In this example we apply the operator '+' to the numbers 3 and 4 and achieve the result 7.

Every number is distinct and can belong to a given set of numbers. There are three familiar sets of numbers that we encounter in everyday life. These are the set of Natural numbers, the set of Integers and the set of Real numbers.

What is contained in each of these sets?

Firstly Natural numbers are the first numbers that we all learn to count with.
0, 1, 2, 3, etc. It is the set of all positive whole numbers greater than or equal to zero.

Secondly Integers, like natural numbers are also whole numbers but they also include negative numbers in addition to the positive ones, e.g. -2, -1, 0, 1, 2, etc.

Finally the set of Real numbers contains all the integers but also all the decimal numbers in between the whole numbers, e.g. 1.1, 1.2, 1.3, 2, etc. The set of Real numbers is probably the most well known set due to its relationship to currency denominations. € 1 = 100¢, therefore we can say that 1¢ = 0.01 of a Euro.

Now that we have identified the sets of numbers that we deal with in everyday life we must look at how we use them. How do we decide the value or meaning of a number?

Like most things in life we need an order so that things make sense. Numbers too have an order and we first learned the order of numbers by number lines. A number line enabled us to give structure to an otherwise mixed up set of numbers and also showed us that the relative value of each number. For example the number 3 comes before 5 on the number line therefore we conclude that 3 is less than 5.

1_______2_______3_______4_______5_______6

Fig 1.1 the number line

Having identified that 3 is less of a value than 5, we now recognise another area of numbers known as inequalities. An inequality, as the name suggests is something that is not at the same level, there is a difference between two things. In math we know this to be represented by the use of greater than and less than signs, e.g $3 < 5$ or $5 > 3$.

1.1.2 Review of Basic Algebra

The real number system and number line

· Natural numbers
· Whole numbers
· Integers
· Rational numbers
· Irrational numbers

Ordering the Real Numbers

$a < b$ if $b - a$ is positive

Bounded Intervals on the Real Number Line

· $[a,b]$
· (a,b)
· $[a,b)$
· $(a,b]$

Definition of Absolute Value

If *a* is a real number, then its absolute value is given by

$|a| = a$ if $a \geq 0$

or

$|a| = -a$ if $a \leq 0$

Definition of an Algebraic Expression

An ***algebraic expression*** is a collection of letters **(variables**) and real numbers **(constants**) combined using the operations of addition, subtraction, division and multiplication.

Basic Rules of Algebra

Commutative Rule of Addition	a+b = b+a
Commutative Rule of Multiplication	ab = ba
Associative Rule of Addition	(a + b) + c = a + (b + c)
Associative Rule of Multiplication	(ab)c = a(bc)
Additive Identity Property	a + 0 = a
Multiplicative Identity Property	(a)1 = a
Additive Inverse Property	a + (-a) = 0
Multiplicative Inverse Property	(a)(1/a) = 1
Distributative Property	a(b + c) = ab + ac

Properties of Negation

- (-1) a = - a
- -(-a) = a
- (-a) b = - (ab) = a (-b)
- (-a) (-b) = ab
- -(a + b) = (-a) + (-b)

Properties of Zero

- a + 0 = a
- (a)0 = 0
- 0 / a = 0 if a ≠ 0
- a / 0 is undefined
- if ab = 0 then either a = 0 or b = 0

Properties of Fractions

Equivalent Fractions	$\frac{a}{b}=\frac{c}{d}$ then $ad=bc$
Rules of Signs	$-\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}$ and $\frac{-a}{-b}=\frac{a}{b}$
Generate Equivalent Fractions	$\frac{a}{b}=\frac{ac}{bc}$ where $c\neq 0$
Add or Subtract with Like Denominators	$\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$
Add or Subtract with Unlike Denominators	$\frac{a}{c}\pm\frac{b}{d}=\frac{ad\pm bc}{cd}$
Multiply Fractions	$\frac{a}{c}\cdot\frac{b}{d}=\frac{a\cdot b}{c\cdot d}$
Divide Fractions	$\frac{a}{c}\div\frac{b}{d}=\frac{a}{c}\cdot\frac{d}{b}=\frac{a\cdot d}{c\cdot b}$

Exponents

Let a be a real number, a variable, or an algebraic expression, and let n be a positive integer. Then

$$a^n = a \times a \times a \ldots a$$

where n is called the exponent **(or power)** and a is called the base.

Properties of Exponents

- $a^m a^n = a^{m+n}$
- $\dfrac{a^n}{a^m} = a^{n-m}$
- $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$
- $a^0 = 1$
- $(ab)^n = a^n b^n$
- $(a^n)^m = a^{nm}$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Roots and Rational Exponents

Let a, b be real numbers and n >= 2 be a positive integer. If

$a = b^n$ then b is the n^{th} - root of a

This is generally denoted by

$$b = \sqrt[n]{a}$$

Properties of Roots

- $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
- $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
- $\left(\sqrt[n]{a}\right)^n = a$
- $\sqrt[n]{a^n} = \pm a$

If a is a real number and n is a positive integer such that the principal n^{th} - root of a exists, then we define

$$a^{1/n} = \sqrt[n]{a}$$

Moreover if m is a positive integer that has no common factor with n then

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$$
$$= \left(a^m\right)^{1/n} = \left(\sqrt[n]{a^m}\right)$$

1.1.3 Equations

An equation consists of two algebraic expressions and an equals sign (=) between them. e.g. 2x + 2 = 4. In order to solve this equation we must solve for x, that is to say we must discover the value of the unknown quantity x in the equation. To do this we bring all the unknowns to one side and all the known values to the other, remembering that if we switch a value from one side of the equals to the other we negate it.

So our equation now becomes

$2x = 4 – 2$ or $2x = 2$, giving us $x = 1$

So now we can solve equations where we have one unknown, generally x.

1.2 The Straight Line

The straight line is one of the most fundamental areas of geometry. Everywhere you look there are straight lines that we use in our lives. We use it to measure the length of a room and to find out the distance between two points *"as the crow flies"* on a map.

If we take this straight line and put it into a mathematical environment, we develop a new view of it. Lines in math all have certain qualities such as length, start point, end point, direction, slope, etc. How do we calculate these attributes of a line?

1.2.3 The Coordinate Plane

In order to do any calculations regarding lines we must familiarise ourselves with what is known as the co-ordinate plane. Put simply this is no more than a grid like on any map. We have two axis that we are concerned with. The *x axis*, which is a horizontal line and the *y axis* which is a vertical line. These two lines intercept each other at *right angles* and the *point* at which they meet is referred to as the *origin*.

The origin is a special point within the plane because it is a reference point for all other points within the plane. Just like a number line both of the aforementioned axis extend to a point in both directions. Starting at one value and continuing to another some distance away. See example below.

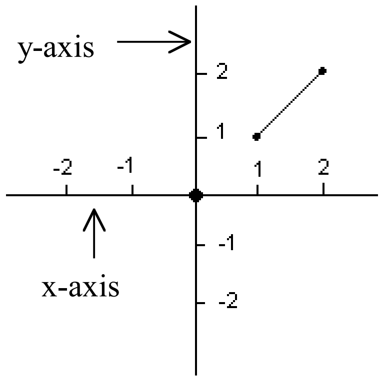


Fig 1.2 Coordinate plane

In the above diagram we can see our co-ordinate plane. We can see the origin where the two lines meet and we can see two points in the plane and a line joining them together. We can also see the x and y axis clearly marked.

Now if we follow our x axis along we will see that the first point is directly above the 1 and the second point is directly above the 2. This means that we have what are known as x-coordinates for both our points.

Then we follow our y-axis upwards and find out that the first point is directly opposite the 1 and the second point is directly opposite the 2. So as you might guess this gives us y-coordinates for both our points. Now combining both the x and y coordinates for each point we can establish an absolute reference for each point, i.e. the point in the coordinate plane where the dot is located.

e.g. point 1 = (1, 1) and point 2 = (2, 2)

We use the bracket notation when writing points, (x-coordinate, y-coordinate).

1.2.4 Length of a Line

Now that we have looked at the basis for locating and describing points within the coordinate, let us look at the line joining these two points. We can see in the diagram that a straight line joins point 1 and point 2, starting at point 1 and ending at point two.

How do we calculate the length of this line?

It's now time to introduce the first formula, the length of a straight line.
As we see a line is made up of two points. Each point has a coordinate location (x, y). When we are dealing with a line we will give each point a distinct alias, the convention is (x_1, y_1) for point 1 and (x_2, y_2) for point 2.

$$\text{length of line} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So let's put this new formula into practice and find out the length of our line.

(x_1, y_1) = (1, 1)
(x_2, y_2) = (2, 2)

$$\begin{aligned}\text{length of line} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-1)^2 + (2-1)^2} \\ &= \sqrt{(1)^2 + (1)^2} \\ &= \sqrt{2}\end{aligned}$$

1.2.5 Slope of a Line

Now that we have learned how to calculate the length of a line we now must learn how to calculate the slope of the line. Again this is all based on coordinates. We can pick any two points on a line in order to find the slope of it. In our example the only two points that we have identified are the start and end points of the line. So we will use these two points again.

Time for another formula!!

$$\text{slope of a line} = \frac{y_2 - y_1}{x_2 - x_1}$$

So, again we will apply this new formula to our line and find out its slope.

$$\begin{aligned}\text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-1}{2-1} \\ &= 1\end{aligned}$$

So we have now calculated the slope of our line to be 1. This means that for every 1 unit you travel along the x-axis our line will rise by 1 unit up the y-axis. So we spot a trend, our first point was (1, 1) and our second point was (2, 2) therefore applying our newly found knowledge we can guess that if we continued our line on further we would encounter a point (3, 3). Try to view it as a stairs, for every step you take up the stairs you climb higher but also travel inwards. If you are on the third step then you are 3 steps high and 3 steps in. The slope of a line is often referred to as m.

1.2.6 Midpoint of a Line

Another attribute of a line is its midpoint, the centre of the line. Why would we need to calculate this? Imagine you were given a line in the coordinate plane and told that this was the diameter of a circle. Now you are asked to draw that circle but you cannot measure the line with a ruler to find the centre point of the circle. You do however know from the coordinate plane the start and end points of the line. So by using the midpoint formula we can find our centre point and then draw our circle with a compass from this point.

$$\text{Midpoint of a line} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

So applying this formula to our line we get:

$$\begin{aligned} \text{Midpoint} &= \left(\frac{2+1}{2}, \frac{2+1}{2}\right) \\ &= \left(\frac{3}{2}, \frac{3}{2}\right) \\ &= (1.5, 1.5) \end{aligned}$$

We now know that the midpoint of our line is (1.5, 1.5) and we can draw a circle around it from here.

1.2.7 Equation of a Line

Okay so far we've learned to calculate the length, slope and midpoint of a line. The one important area left is how to calculate what is known as the equation of a line. The equation of a line is what people working with coordinate geometry use to describe a line and extensions of it.

Some of the information that we have already is used in calculating the equation of a line. The ***slope*** is one value that is used to describe a line so it therefore appears in the equation of a line. Another thing that we have looked at is the coordinate plane and the x and y-axis. Now try to imagine a line in the coordinate plane that crossed the y-axis. This line would have a point of intersection with the y-axis, this is known as the ***y intercept***.

So we have the two main things that we need for our equation of the line. The equation of a line takes the form:

$$y = mx + c$$

y is a y coordinate
x is an x coordinate
m is the slope of the line
c is the y intercept, where the line crosses the y-axis

However let's imagine that we do not know exactly where the line crosses the y-axis. Luckily there is also another formula for finding the equation of a line. To use it we still require some of our previously attained knowledge. We need the slope and a single point on the line. This formula takes the form:

$$y - y_1 = m (x - x_1)$$

y_1 is the y-coordinate of the point
x_1 is the x-coordinate of the point
m is the slope of the line

Following is a summary of lines and some exercises to accompany the section.

Solving Equations

An *equation* consists of two algebraic expressions with an equals sign (=) between them.

Aim:

Isolate the unknown quantity, generally x, on one side of the equals sign.

Use:

$+, -, \times, \div, \wedge$. Whatever is done to one side of the equation must be done to the other.

Examples 1

1. Solve $5x+7=4$ for x.

2. Solve $3-4t=7$ for t.

3. Solve $(a+b)^2=16c^2$ for a.

Exercises 2

1. Solve $3x+8=2$ for x.

2. Solve $x^2-2=x^2+4x+8$ for x.

3. Solve $\sqrt[3]{xy}=z$ for x.

Straight Lines.

The simplest equation, and the most important, is the linear equation given by

$$y=mx+c$$

where m and c are known constants.

m is known as the slope

c is called the y-intercept.

This equation when plotted gives a straight line. The easiest way to draw a given line is to find two points on it and draw the line that goes through them.

Examples 3.

Draw the lines given by

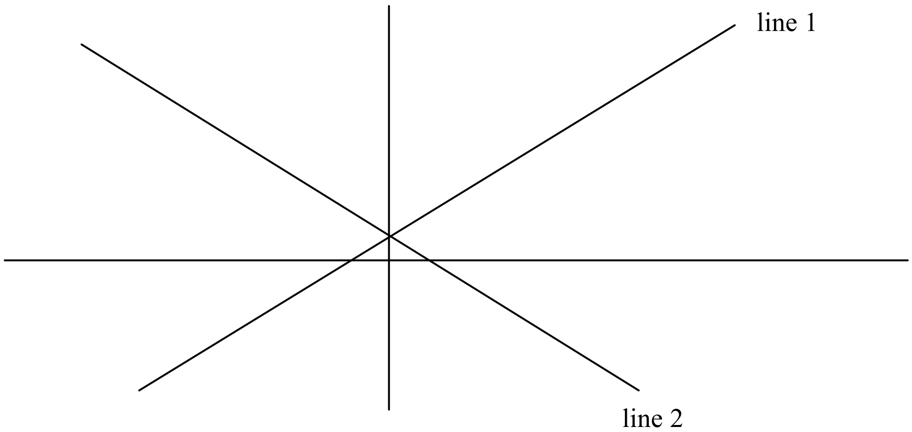
1. $y=2x+4$.
2. $y=x-2$.
3. $y=-x-2$.
4. $y=-3x+1$.

Exercises 4.

Draw the lines given by

1. $y=x+1$.
2. $y=-2x+5$.
3. $2y=6x+4$.

The *slope* of a line is a measure of how quickly it falls or rises. Two lines with the same y-intercept but different slopes cut the y-axis at the same point but rise or fall at different rates.

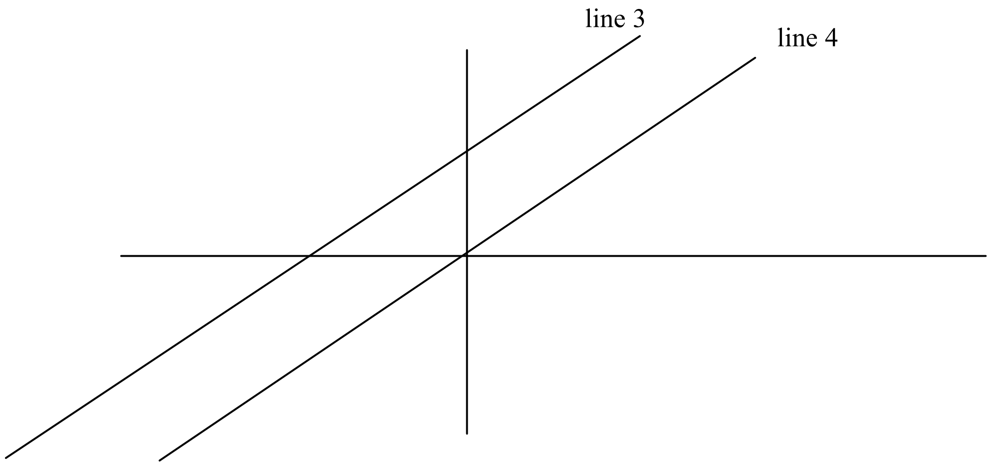


It turns out in this diagram that line 1 has a positive slope, since it is rising and that line 2 has a negative slope since it is falling. Symbolically this means that

$m>0$ for line 1

$m<0$ for line 2

If two lines have the same slope, but different y-intercept then they are parallel.



In this graph line 3 has a positive y-intercept and line 4 has a negative one. They both rise at the same rate and so have the same slope. Clearly they are parallel.

If two lines have the same slope and the same y-intercept then they are the same line, for example

$$y = 2x + 4$$
$$-3y = -6x - 12$$

The slope of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

Examples 5.

Draw the line and find the slope of the line through the points

1. $(0,1)$ and $(2,3)$
2. $(3,-1)$ and $(-2,2)$

Exercises 6.

Draw the line and find the slope of the line through the points

1. $(2,1)$ and $(3,2)$
2. $(-1,1)$ and $(1,-3)$
3. $(-2,-3)$ and $(0,-1)$

If we know the $y-$intercepts in the above examples then we could write down the equations of the lines. In general we don't have this information so we must use the following formula.

$$y - y_1 = m(x - x_1)$$

where (x_1,y_1) is either of the points given.

Examples 7.
Find the equations for the lines given in examples 5.

Exercises 8.
Find the equations for the lines given in exercises 6.

The equation of a line in standard form is

$$ax+by=c$$

where a, b and c are known constants.

Graphing straight lines.
To graph a straight line all we need is two points on the line. The easiest way to do this is as follows.

1. Let $x = 0$ and solve the equation for y
2. Let $y = 0$ and solve the equation for x.

Alternatively if there is a point which is obviously on the line then we could use this, e.g. the point $(1,1)$ is on the line $x + y = 2$.

Example 9.

1. Graph the line $y+2x=4$.
2. Graph the line $3y+5x=2$

The following example is akin to what we will be doing later on in the course.

Example 10 (linear modeling)

The total amount of advertising expenses (in billions of dollars) in the US from 1980 to 1988 is given by

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988
Exp.	54.78	60.43	66.58	75.85	88.10	94.75	102.1	109.7	118.3

A linear equation that approximates this data is given by

$$y = 8.1377\,x + 53.029$$

where x is the number of years after 1980. Plot the data and check this model for accuracy.

Example 11.

The cost of hiring a car is € 35 plus € 1 for every mile over 100 miles. Give an expression for the cost of a trip of x miles, in a rented car when $x>0$.

Example 12.

I wish to buy pens and paper for a class. A pen costs 20c and a refill pad costs € 1.20. I have € 5 to spend. Express this situation as a linear equation. If I buy 3 refill pads, how many pens can I buy?

Simultaneous Equations.

Two straight lines can meet in one of two ways; either they lie on top of the other or they meet in one point. The only other alternative is that they are parallel, they do not meet at all.

Finding where two lines meet, if at all, involves solving two equations simultaneously.

Example 13.

Graph the following two lines and find their point of intersection.

$$3x + 2y = 5$$
$$4x - 3y = 2$$

There are two main analytical methods to solve this problem. They are the methods of elimination and substitution.

The Method of Elimination.

- Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen nonzero constants.
- Add the equations to eliminate one variable and solve the resulting equation.
- Back-substitute the value just obtained into either of the original equations and solve for the other variable.
- **Check your solution** in both of the original equations.

Examples 14.

Solve the following equations for x and y, using the method of elimination.

1. $$\begin{aligned}2x+3y&=-7\\3x+y&=-5\end{aligned}$$

2. $$\begin{aligned}5x+3y&=9\\2x-4y&=14\end{aligned}$$

Exercises 15.

Solve the following equations for x and y, using the method of elimination.

1. $$\begin{aligned}4x-5y&=7\\3x+2y&=5\end{aligned}$$

2. $$\begin{aligned}x-2y&=1\\3x+4y&=4\end{aligned}$$

Draw the pairs of lines given above and in examples 13 on four pairs of axes. Check if the graphical and analytical solutions agree.

The Method of Substitution.

- Solve one of the equations for y in terms of x, or vice versa.
- Substitute this value into the other equation and solve for the other variable.
- Use this value in one of the original equations and solve for the other variable.
- **Check the answer**.

Examples 16.

Solve the following equations for x and y, using the method of substitution.

1. $$\begin{aligned} 2x+3y&=-7 \\ 3x+y&=-5 \end{aligned}$$

2. $$\begin{aligned} 5x+3y&=9 \\ 2x-4y&=14 \end{aligned}$$

Exercises 17.

Solve the following equations for x and y, using the method of elimination.

1. $$\begin{aligned} 4x-5y&=7 \\ 3x+2y&=5 \end{aligned}$$

2. $$\begin{aligned} x-2y&=1 \\ 3x+4y&=4 \end{aligned}$$

Draw the pairs of lines given above and in examples 15 on four pairs of axes. Check if the graphical and analytical solutions agree.

Exercises 18.

Using both of the methods just outlined solve the following equations for the corresponding variables.

1. $$\begin{aligned} 2x+3y&=5 \\ 3x-4y&=3 \end{aligned}$$

2. $$\begin{aligned} 7x+2y&=5 \\ x-6y&=5 \end{aligned}$$

Example 19. (Driving Distance)

In a trip of 300 miles two people do the driving. One person drives three times as far as the other. Find the distance that each person drives.

Example 20. (Ticket Sales)

500 tickets were sold for a play. The tickets for adults were €7.50 and those for children were €4.50. The total receipts for the show was €3,300. How many tickets of each kind were sold?

Matrices and Linear Equations.

In solving systems of equations the important factor has been the coefficients of the variables.

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + y &= 5 \end{aligned} \qquad \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$$

The array of $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$ is called a 2×2 matrix. In general if m and n are positive integers, then an $n\times m$ matrix (read "n by m") is a rectangular array

$$\begin{pmatrix} a_{11} & a_{12} & \cdots a_{1m} \\ a_{21} & a_{22} & \cdots a_{2m} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots a_{nm} \end{pmatrix}$$

This method of concentrating on the "matrix of coefficients" has many benefits

- we concentrate on what is really important
- ease of use and computation

A matrix with only one row(column) is called a row(column) matrix or vector. We shall concentrate on 2×2's.

Before dealing with solving linear equations using matrices we shall look at the algebra of matrices.

Matrix Addition

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

multiplication by scalars.

$$t\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ta & tb \\ tc & td \end{pmatrix}$$

Examples 22.

Evaluate

1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & -3 \\ 1 & -7 \end{pmatrix}$
2. $\begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 2 & -4 \\ 3 & -7 \end{pmatrix}$
3. $3\begin{pmatrix} 2 & -1 \\ -2 & 3 \end{pmatrix} - 2\begin{pmatrix} 3 & 4 \\ -1 & 6 \end{pmatrix}$

Matrix Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

Examples 23.

Evaluate

1. $\begin{pmatrix} 4 & 7 \\ -3 & 8 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ 3 & 4 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

Inverse of a Matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Examples 24.

Find the inverse of

1. $\begin{pmatrix} 1 & 4 \\ 3 & 8 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 7 \\ 3 & 10 \end{pmatrix}$

3. $\begin{pmatrix} -2 & 5 \\ -4 & 8 \end{pmatrix}$

Multiplication of a matrix by a vector is a simplification of what we have already seen.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\,x + b\,y \\ c\,x + d\,y \end{pmatrix}$$

<u>Examples 25.</u>
Multiply

1. $\begin{pmatrix} -2 & 5 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

2. $\begin{pmatrix} 2 & -5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

We call the matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

the identity matrix, since $IA = A = AI$ when A denotes any 2×2 matrix. In addition

$$AA^{-1} = A^{-1}A = I .$$

We will use this idea in solving systems of linear equations.

Let a, b, c, d, e, f be constants. The system of linear equations

$$a\,x + b\,y = e$$
$$c\,x + d\,y = f$$

can be given by the following matrix representation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

<u>Example 26.</u>

Solve the following system of equations.

1. $$\begin{aligned} 2x + 3y &= 7 \\ 4x + 5y &= 2 \end{aligned}$$

2. $$\begin{aligned} 3x - 5y &= -2 \\ 2x + y &= -3 \end{aligned}$$

3. $$\begin{aligned} x &= 5y + 6 \\ y - 4 &= 2x \end{aligned}$$

Note: Matrix multiplication is not commutative.

$$\begin{pmatrix} -2 & 5 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \neq \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 5 \\ -4 & 8 \end{pmatrix}$$

We shall see, by using excel, that systems of equations in a larger number of unknowns can be solved by using matrices.