
National Certificate in Computer Applications Year 1

Quantitative Methods (ACCS) Summer Examination 2001

Date : May, 2001

Duration : 2 hours

Examiner : Jonathan Brazil

Instructions to candidates :

- **Answer five questions of your choice**
 - **Attempt each part of each question you answer**
 - **All questions carry equal marks**
 - **Write clearly**
 - **Read the questions carefully**
 - **Show all rough work**
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Question 1

- (a) Draw the following lines using the same axes for each
- (i) $5x - y = 17$
 - (ii) $6y - 2x = 10$
- (6 marks)
- (b) Where do these lines intersect? (write your answer in the correct form)
- (4 marks)
- (c) A bridge is 1 mile long. If a jogger is crossing the bridge and covers the first quarter of the bridge at 4 m.p.h., the next two quarters of the bridge at 3 m.p.h., the next eighth of the bridge at 5 m.p.h. and the final eighth of the bridge at 4 m.p.h. How long will it take the jogger to cross the bridge? (the formulae at the back of the paper may assist you in answering this)
- (4 marks)
- (d) Which of the following lines are increasing? (i.e. have a positive slope)
- (i) $4x - y = 3$
 - (ii) $3x - 2y = 4$
 - (iii) $2x = -2y - 2$
- (6 marks)

Question 2

- (a) Perform the following matrix operations (show all workings)
- (i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} -2 & -4 \\ 3 & 6 \end{pmatrix} =$
 - (ii) $\begin{pmatrix} 8 & 0 \\ -4 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 5 \\ -3 & 2 \end{pmatrix} =$
 - (iii) $2 \begin{pmatrix} 2 & 4 \\ -1 & 2 \end{pmatrix} + 3 \begin{pmatrix} 1 & 5 \\ -3 & 4 \end{pmatrix} =$
 - (iv) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 5 & 9 \end{pmatrix} =$
- (8 marks)
- (b) Find the inverse of $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$
- (6 marks)
- (c) Hence solve the simultaneous equations below using matrices
- $$\begin{aligned} 2y &= -3x + 9 \\ 4y &= x - 15 \end{aligned}$$
- (6 marks)

Question 3

(a) Explain the term effective rate of interest. (4 marks)

(b) Fred is buying a house as a long term investment. He already has £25,000 which he can use as a deposit. The cost of the house will be £118,000. Assuming that he gets a mortgage for the remainder of the cost of the house from his bank at a rate of 5.2% per annum, compounded monthly. His two options for the duration of the mortgage are:

- (i) 15 years
- (ii) 20 years

A financial wizard has told Fred that his house will be worth £220,000 in 15 years and £260,000 in 20 years. Given that Fred plans to sell the house when the mortgage is repaid, which option will give him the greatest profit margin? (8 marks)

(c) Catherine wishes to go on holidays in 12 months time to Hawaii. The holiday will cost her £3,200. She currently has £2,450 in savings that she can put away for 12 months into a special savings account with a healthy 5.75% interest, compounded monthly. If she takes this option will her savings be enough to send her to Hawaii in 12 months time? (8 marks)

Question 4

(a) Describe the declining balance rate of depreciation. (4 marks)

(b) Mary bought a car 7 years ago for £10,500. The lifespan of the car is 12 years and the depreciation is said to be at **twice** the declining balance rate. She wishes to buy a new car in 3 years using her current car as a trade in. Her current car would be traded in at its book value at the time. How much will she need to put away now into an account paying 4.35%, compounded monthly in order to afford the car? (8 marks)

(c) Let's imagine more realistically that Mary will not have this lump sum to put away. How much would she have to put into the same account every month for 3 years in order to afford the car? (8 marks)

Question 5

A factory manufactures two products, A and B. Product A requires 4kg of raw material to manufacture one unit and product B requires 7.5kg of raw material to manufacture unit. Product A takes 10 hours to manufacture and product B takes 4 hours to manufacture. In any given week the total weight of raw material available is 30kg and the total number of working hours available is 54 hours. At most 5 units of product A can be manufactured in a week and at most 4 units of product B can be manufactured in a week. A final constraint for the factory is that for every 4 units of product A they manufacture, they must also produce at least one of product B.

Product A generates a profit of £250 per unit and B generates a profit of £320 per unit.

Let X = number of units of product A made in a week

Let Y = number of units of product B made in a week

Let P = profit made in a week

- (i) Formulate this problem as a linear programming problem.
(8 marks)
- (ii) Find and identify the feasible region and hence determine the production which maximises the factory's profits and calculate this maximum profit per week.
(6 marks)
- (iii) As we can see manufacturing product A takes 10 hours and hence only a maximum of 5 can be made in the available working hours along with 1 unit of product B. If the factory increased the working hours in a week to 64 hours and increased the raw materials available to 32kg allowing a maximum of 6 units of product A to be manufactured and 1 unit of product B. This would come at a cost of £180 to the factory. Recalculating the maximum profit in a week to accommodate these changes, would this be a favourable option for the factory.
(6 marks)

Question 6

The following table is a presentation of the average marks of a sample of students obtained from a survey in a certain Irish college. Examine the data and answer the questions which follow:

70	63	42	40	42
63	70	70	89	84
50	42	80	59	70
63	40	50	60	63

Table 6.1 average marks of students

- (a) Present this data in a frequency distribution table. (2 marks)
- (b) Calculate the mean and median of this data. (6 marks)
- (c) Calculate the inter-quartile range and the standard deviation. (6 marks)
- (d) Give a brief description of multistage sampling. (2 marks)
- (e) Assuming that 8.5% of the population is unemployed and 48% of the population is male. Given this data, construct a stratified sampling plan for a sample of 1500 people. (4 marks)

Question 7

An independent research team have discovered that the average number of heart attacks recorded per year seems to be in some way related to the average amount of tax paid each month by professionals! Could there really be a link? You are presented below with the results obtained from the research, examine them and answer the questions that follow:

# of Heart Attacks	980	1010	870	960	990	890
Tax Paid Per Month	280	283	265	272	281	275

Table 7.1 average number of heart attacks per year and average tax paid per month

- (a) Calculate and draw the regression line that relates the tax paid per month and the number of heart attacks per year. (6 marks)
- (b) Calculate the correlation coefficient. (4 marks)
- (c) Estimate the expected number of heart attacks per year when the average tax paid per month is:
- (i) 270
 - (ii) 300
- Which of these would you regard to be more accurate and why? (6 marks)
- (d) Imagine the average tax paid each month went up by 2%, would you expect the average number of heart attacks to increase by 2% also? Why? (4 marks)

Linear Regression

1. Linear regression parameters **a** and **b**

$$S_{xy} = \frac{\sum xy}{n} - (\bar{x})(\bar{y})$$

$$S_{xx} = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$S_{yy} = \frac{\sum y^2}{n} - (\bar{y})^2$$

$$b = \frac{S_{xy}}{S_{xx}} \quad a = \bar{y} - b\bar{x}$$

2. Correlation coefficient

$$r^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}}$$

Financial Mathematics

1. Future value of annuity

$$I = \frac{R(1+r)((1+r)^t - 1)}{r}$$

2. Present value of annuity

$$A = \frac{R\left(1 - \frac{1}{(1+r)^t}\right)}{r}$$

Statistics

1. Quartiles

$$Q = L + \frac{r}{f}c$$

2. Arithmetic mean

$$\bar{x} = \frac{\sum f(x)}{f}$$

3. Standard deviation

$$s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Speed

1. Speed * Time = Distance